SAE FD&E Fall 2022

Some Thoughts on Application of Residual Stress in Fatigue Methods

Ric Leist
R. Leist Engineering, PLLC
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Overview

- Load Spectra Considerations
- Hysteresis Effects
- Notch Stress/Strain Determination
- Examples
- Questions

Spectrum Considerations

- Load Sources
  - Flight / Operation
    - Gust, Maneuvers (Symm., Asymm.)
  - Ground / Handling
    - Taxi, Landing, Turns, Handling, etc.
  - Aerodynamic (Buffet)
  - Acoustic / Vibration
  - Stores (Inertia, Bomb Ejection)
- Frequency (Need Data or Estimate)
- Notch stress orientation and location
- Preload / Pre-strain

Spectrum Considerations

Requirements:

- Exceedances Converted to Occurrences
- Order into Spectrum Sequence
- Factors to Consider (Saff, Abelkis)
  - Design Life
  - Block Size (1/10 Lifetime)
  - Block Type
    - Lo-Hi, Hi-Lo, Lo-Hi-Lo, Random, Segmented

Convert to Stresses

- Convert Event to Load (Time Dependent)

  M_x = f(M, altitude, GW, N_z, etc.)

- Convert Load to Remote or Notch Stress

$$\sigma_r = C_1P_x + C_2P_y + C_3P_z + C_4M_x + C_5M_y + C_6M_z + C_7 + C_{res}$$

$$\sigma_n = [C_1]P_x + [C_2]P_y + [C_3]P_z + [C_4]M_x + [C_5]M_y + [C_6]M_z + [C_7] + [C_{res}]$$

Max Principle Stress

NOTE:

- Loads are all a function of product performance and capability.
- Stress is a function of structural characteristics / response to load

Stress Coefficients Format

- Form 1 – Linear Constant (Remote σ)

  $C_i = x$

- Form 2 – Bi-linear Constant (Remote σ)

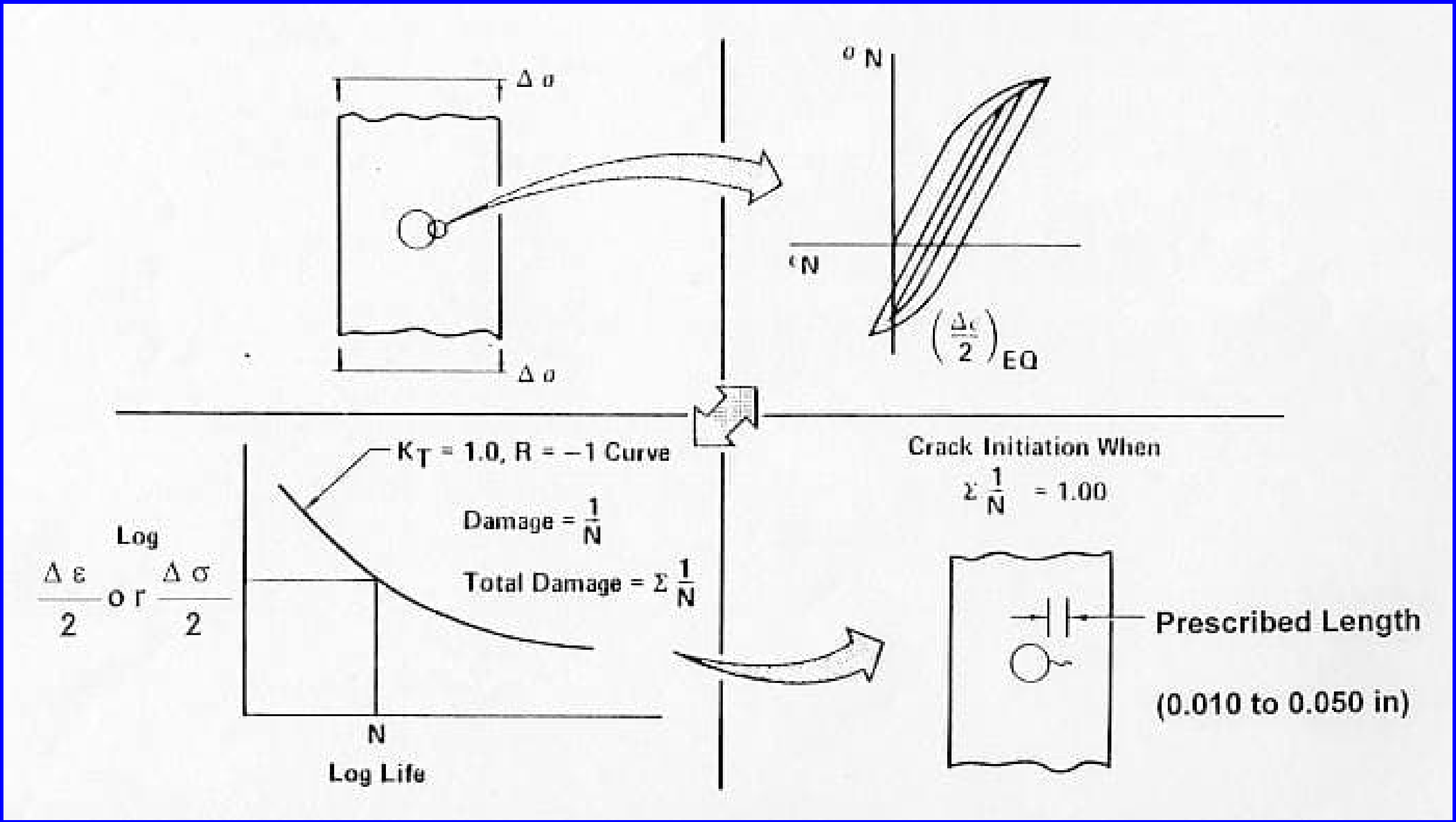
  $C_{i\ tension} = x \quad C_{i\ compression} = y$

- Form 3 – Stress Tensor Matrix

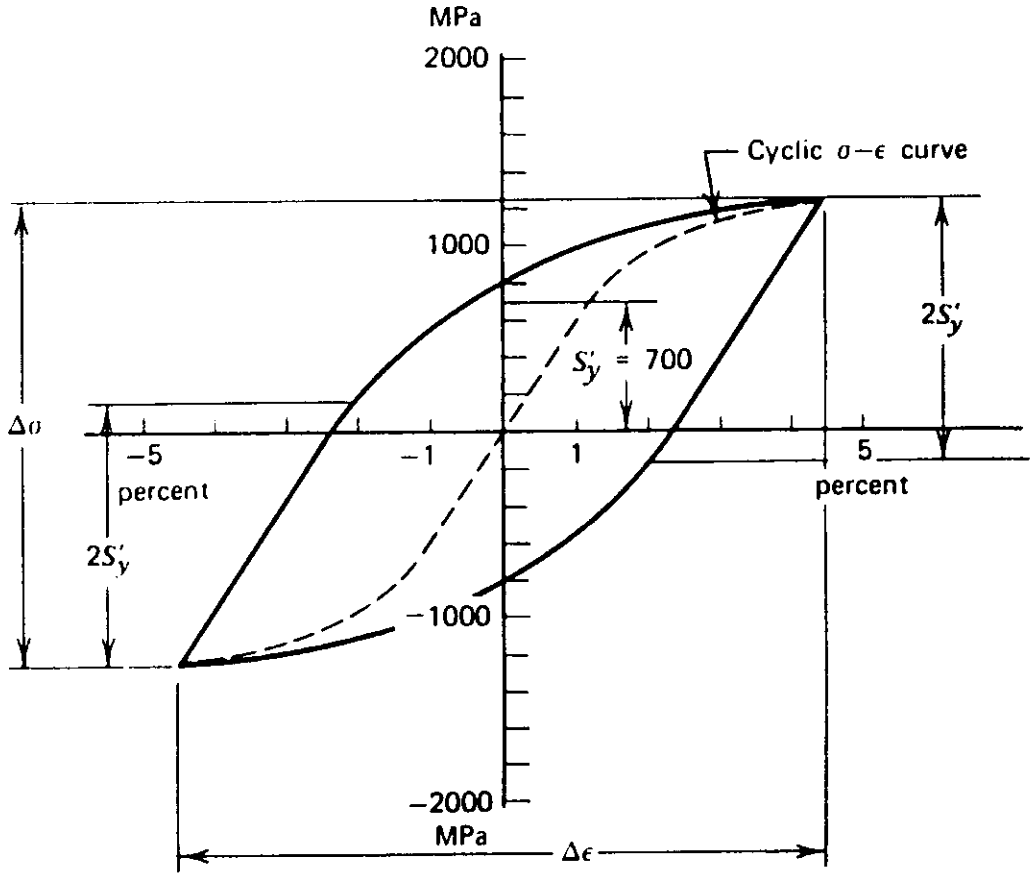
$$C_i = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}_{t \atop c}$$

(Two matrices: Tens and Comp)

Crack Initiation Analysis Method - Nonlinear Notch Stresses



Cyclic Hysteresis σ–ε Data

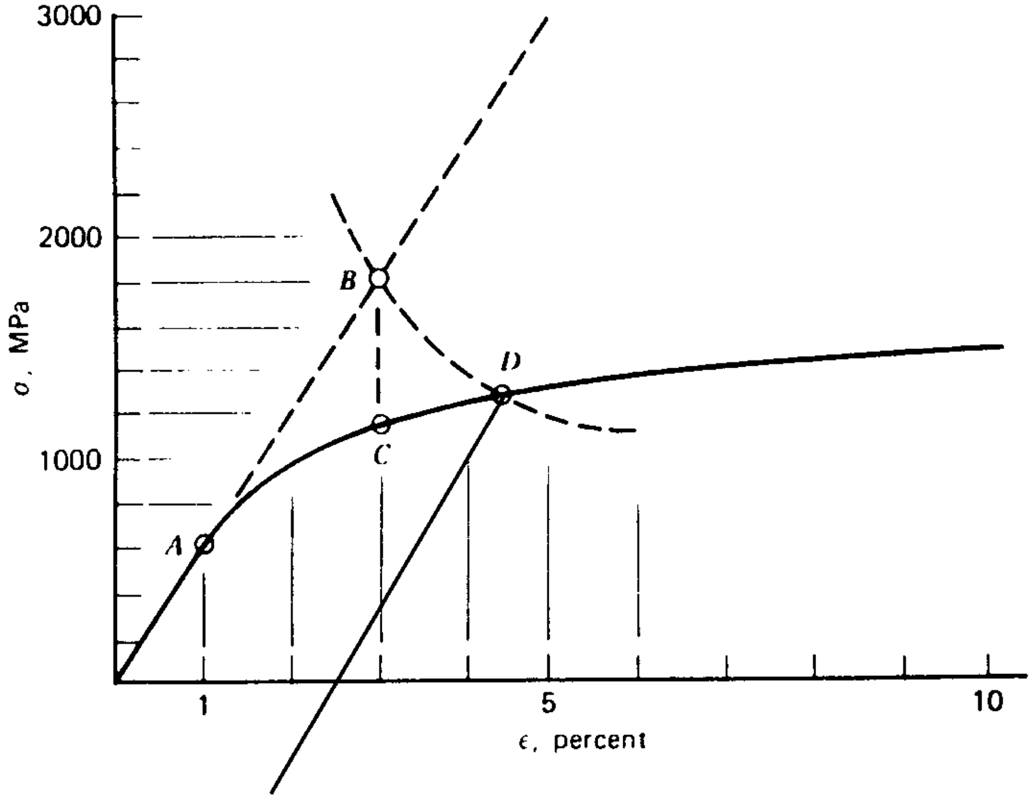


A cyclic stress–strain curve and a hysteresis loop.

REF.: Fuchs & Stephens

Neuber's Rule Stresses

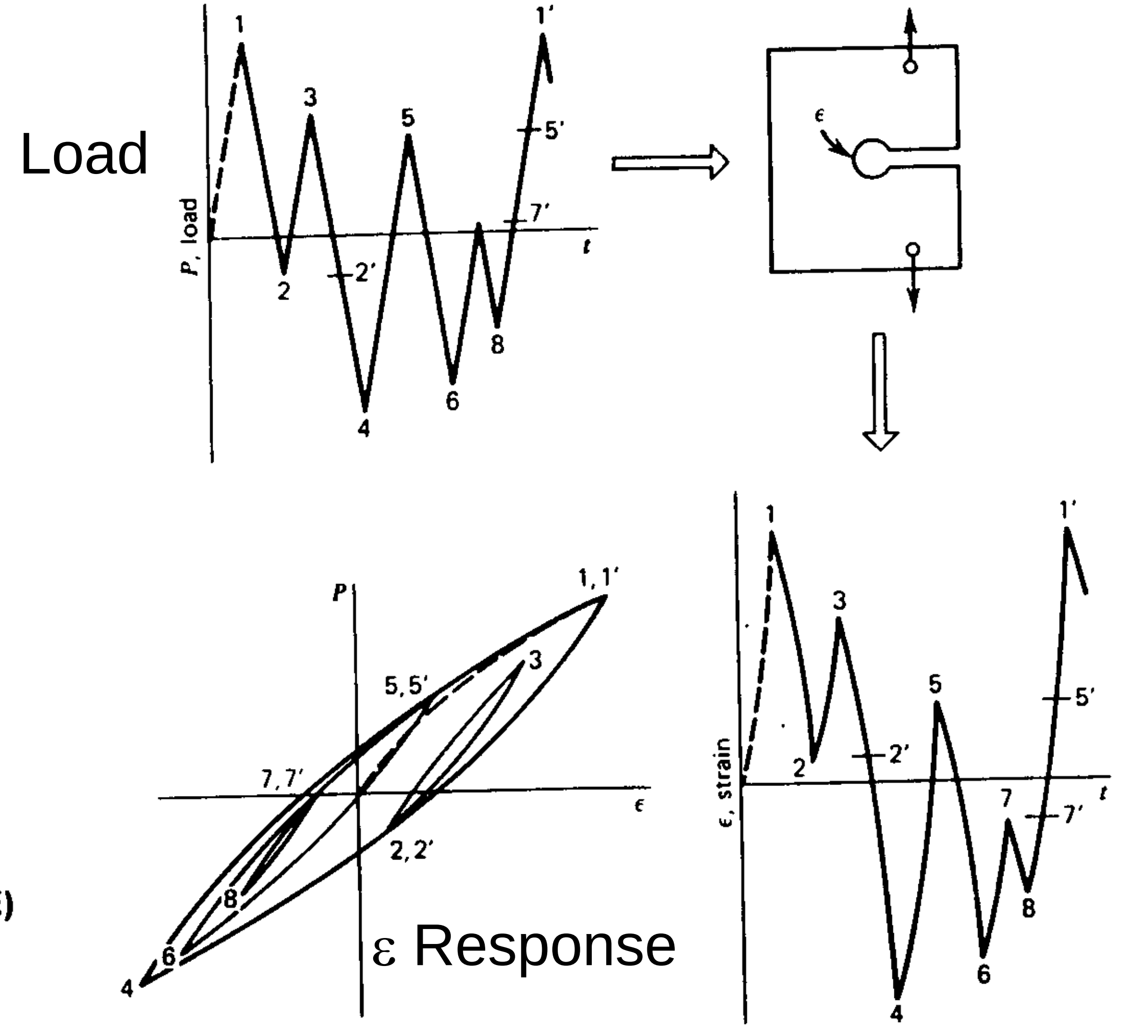
Cyclic Stress vs. Strain



Notch strain determination by the linear rule and by Neuber's rule.

REF.: Fuchs & Stephens

Spectrum Loading



REF.: Fuchs & Stephens (SAE)

Notch Stress Determination

- **Using K_t with Spectrum (Assumes in Phase Loads):**

  $K_t = K_{t\ eq\ gross} = (\sigma_{bp\ max} + \sigma_{bnd\ max} + \sigma_{brg\ max}) / \sigma_{ref\ gross}$

  $K_t = K_{t\ eq\ gross} = (\sigma_{v\ max} + \sigma_{s\ max} + \sigma_{d\ max}) / \sigma_{ref\ gross}$

  $K_t = K_{t\ eq\ gross} = (K_{t\ bp\ gross} * \sigma_{bp\ gross} +$
  $K_{t\ bnd\ gross} * \sigma_{bnd\ gross} + K_{t\ brg\ gross} * \sigma_{brg\ gross}) / \sigma_{ref\ gross}$

- **Using Notch Stress Spectrum (Usually for Out of Phase or Complex Loading):**

  $\sigma_{ne\ max} = \sigma_{bp} + \sigma_{bnd} + \sigma_{brg} + \sigma_{shear} + \sigma_{residual}$

Notch Stress Equation

Neuber's Rule: Uniaxial Plane Stress Form

$$\sigma_n \epsilon_n = K_t^2 \sigma_g \epsilon_g = K_t^2 \sigma_g^2 / E$$

Triaxial Stress Form – Cordes, Glinka, Leist

$$\sigma_n \epsilon_n = \frac{\sigma_1^2}{E}\left[1 - \frac{\mu(\sigma_2 + \sigma_3)}{\sigma_1}\right]$$

Spectrum may be Remote σ or Notch σ

NOTE: $\sigma_n \epsilon_n = \sigma_{ne} \epsilon_{ne}$

Notch Effects - 2 Approaches

- 1. Notch Sensitivity (Only considers σ_1)

$$q = \frac{K_f - 1}{K_t - 1}$$

- 2. Actual Stresses (Cortes, Glinka, Leist)

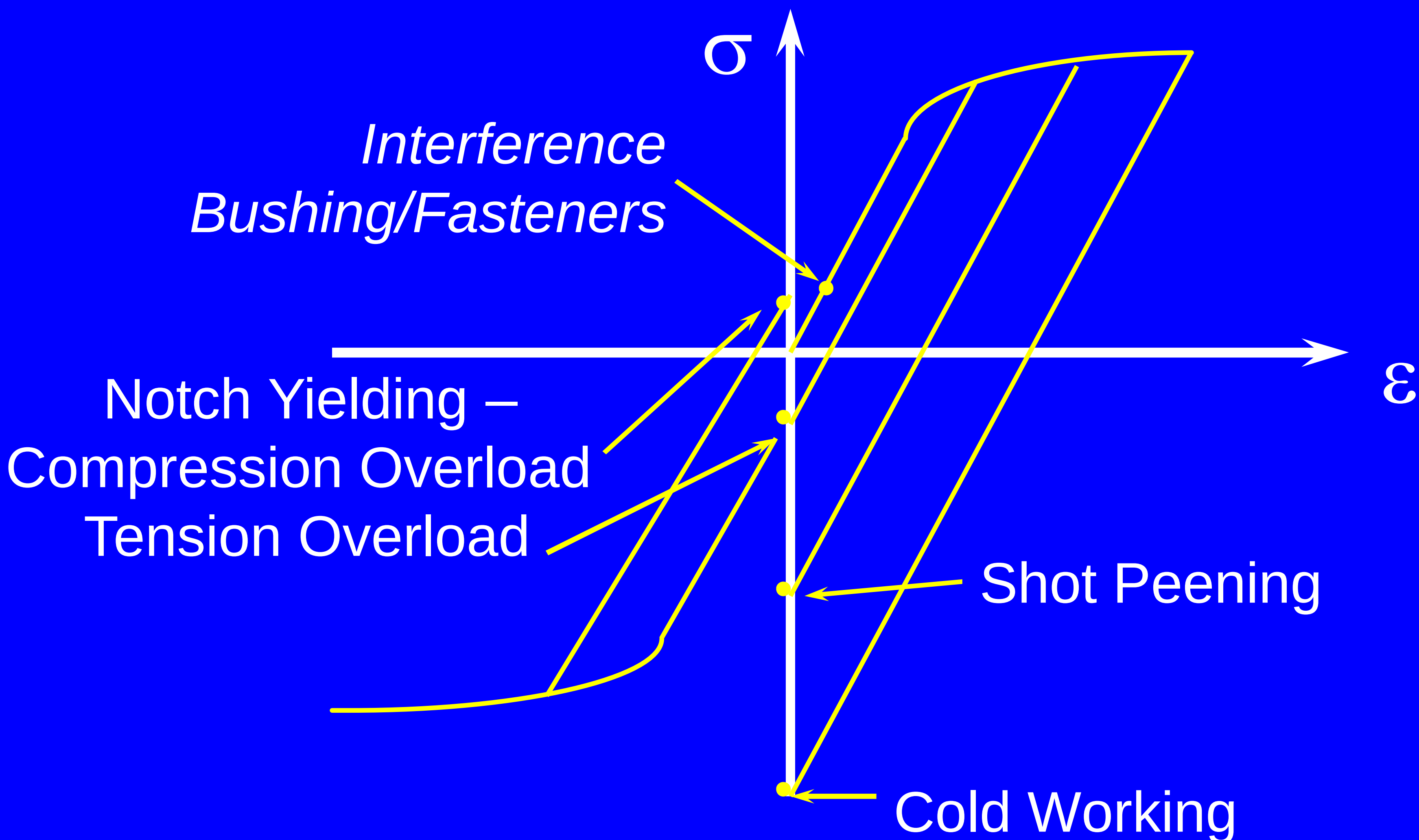
$$\sigma_n \epsilon_n = \frac{\sigma_1{}^2}{E}\left[1 - \frac{\mu(\sigma_2 + \sigma_3)}{\sigma_1}\right]$$

Surface Finish will affect Both Methods

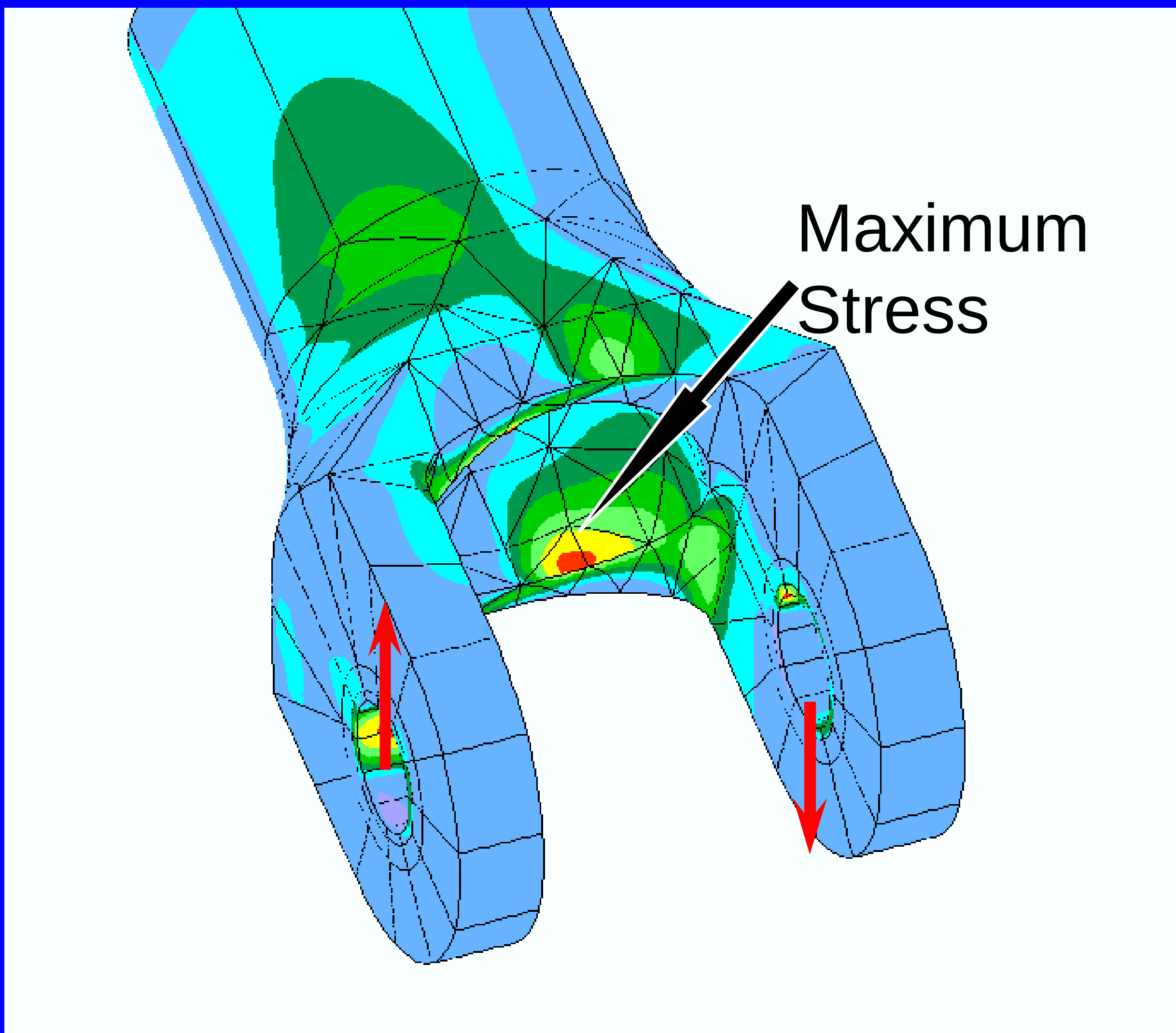
Adjustments to Mean Stress

- Self Stresses
  - Notch Yielding (Tension or Compression)
  - Shot Peening (Compression)
  - Cold Expanded Holes (Compression)
  - Interference Fit (Hoop Tension prior to loading)
  - Process Stresses (e.g. welds, coining, forming)
- Effect is to shift mean of $\sigma - \varepsilon$ curve

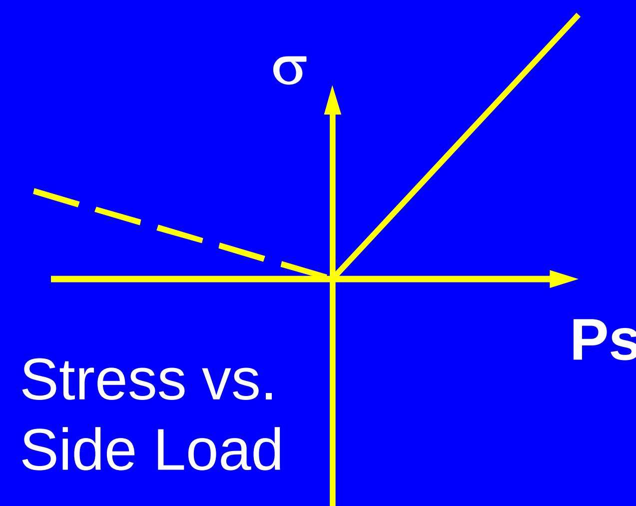
Residual Stress Effects



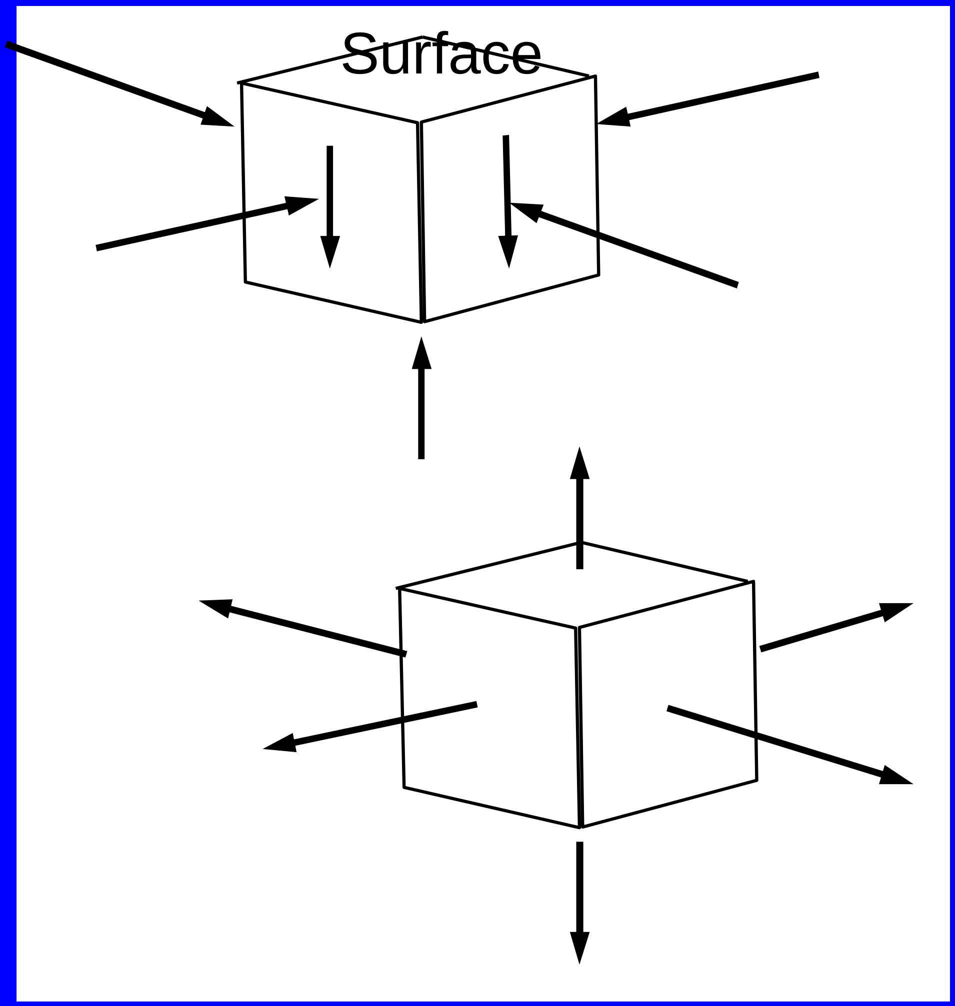
Unit Load Condition Modeling



- **Several Unit Load Conditions May Be Required**
- **Stresses May Be Bilinear with Load**

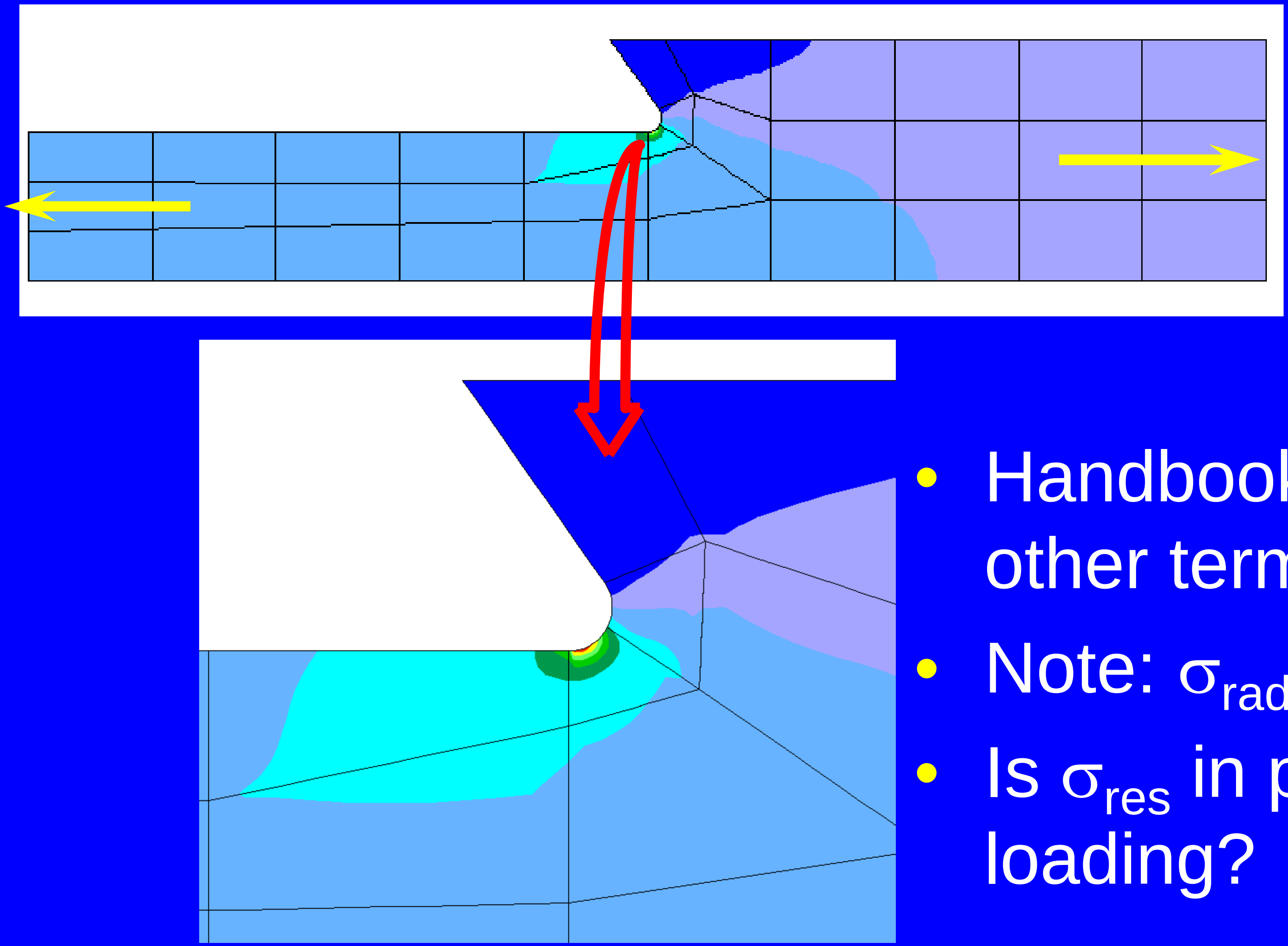


Shot Peen Residual Surface Stresses



- Equiaxed Biaxial Surface Compression
- Subsurface Triaxial Tension (Note: Same for bearing races from bearing loading)

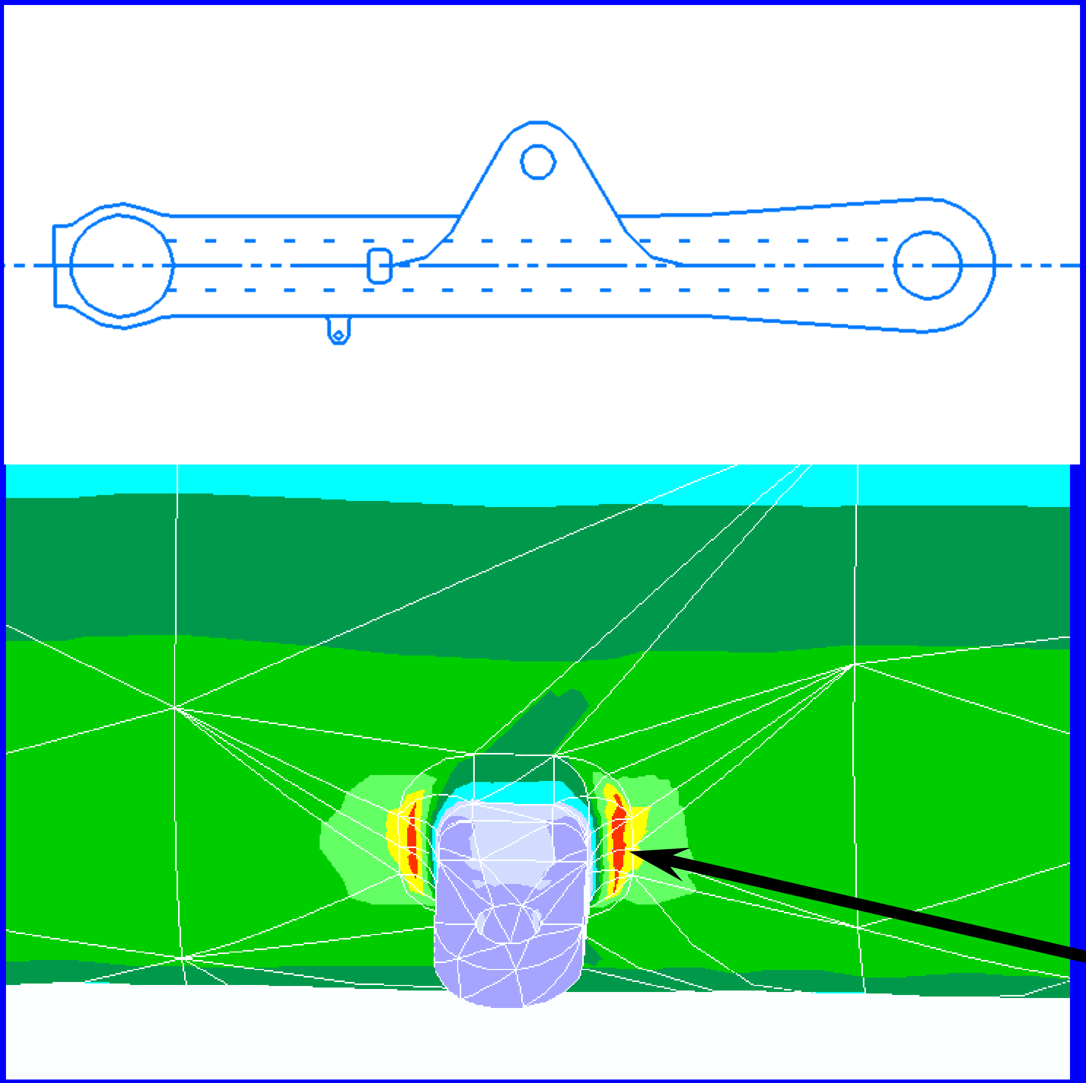
Max Stress Not at Notch Tangency



Restrained Against Vertical Displacement

- Handbook is only σ_1 not other terms
- Note: $\sigma_{rad} = 0$ but $\sigma_z \neq 0$
- Is σ_{res} in phase with loading?

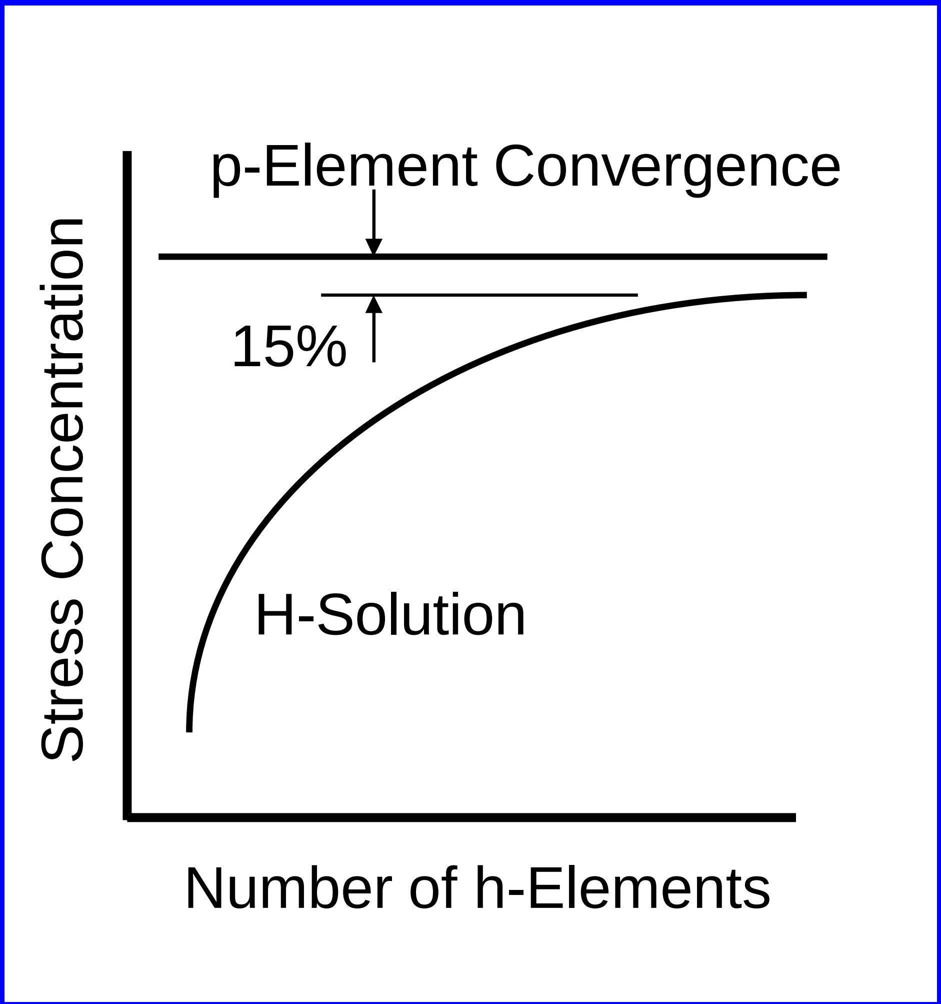
Modeling for Accurate Stress



- Model accuracy is dependent upon type of model and good model practice
  - p-element - approx. 4 elements (p = 8)
  - h-element approx. 90 elements

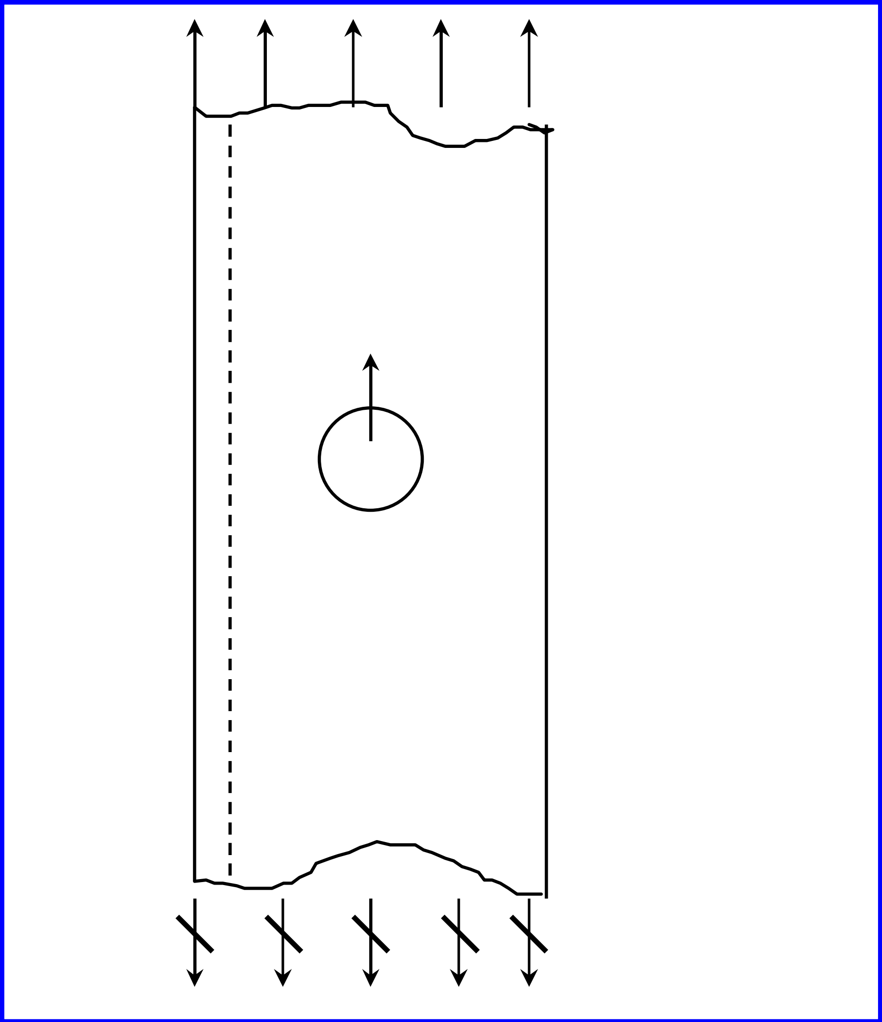
Max Stress

Lug Fillet Convergence with Model Type



- h-Element Modeled to 30 elements in radius
  - Solution difficult to determine multiaxiality
- p-Element Modeled to p-level 8, converged to 1.5%
  - Solution was biaxial, plane strain

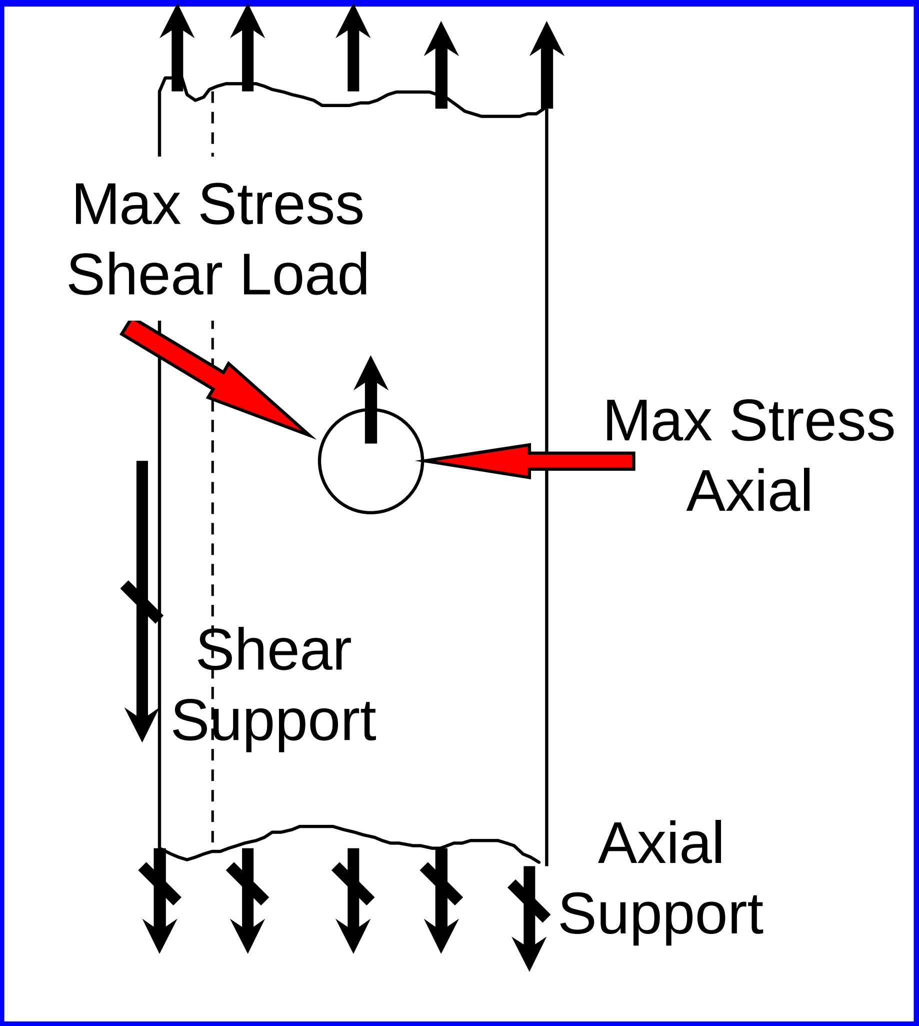
Coldworked Loaded Hole



$$\sigma_{ne\ max} = \sigma_{bp} + \sigma_{bnd} + \sigma_{brg} + \sigma_{shear} + \sigma_{residual}$$

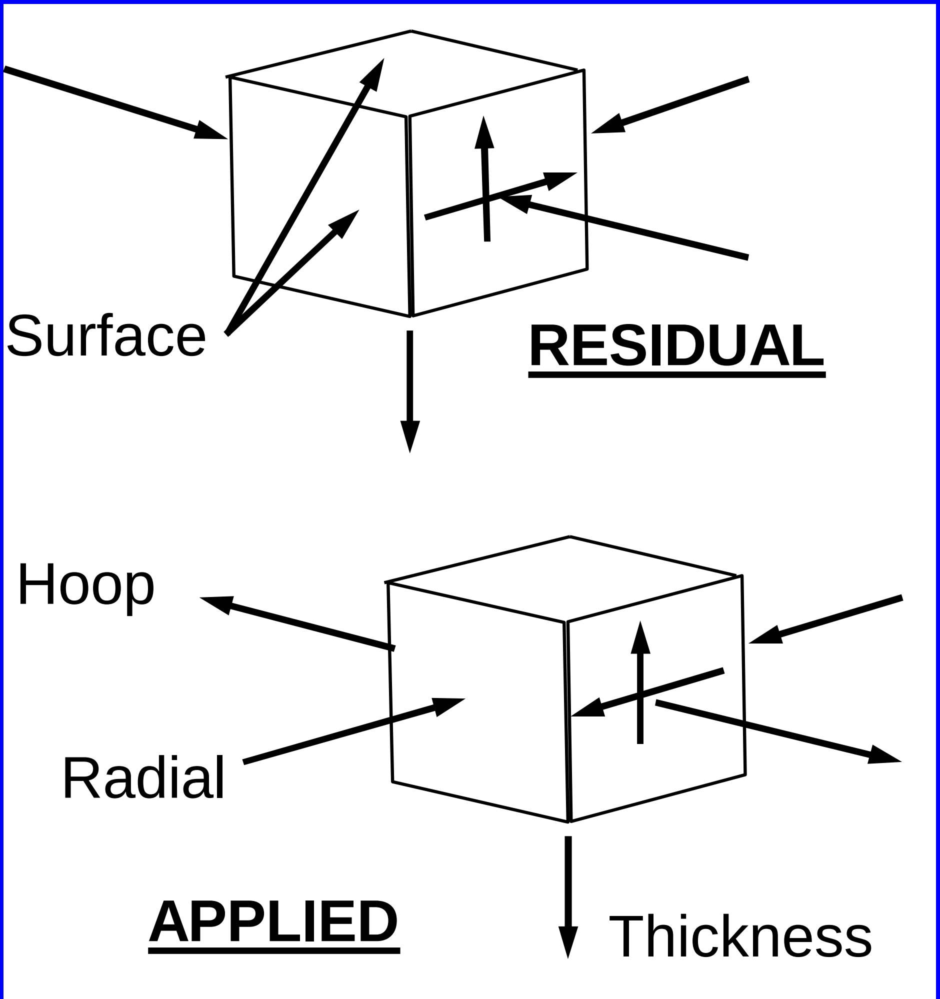
- **Have used successfully in the past but the compressive hoop residual stress is dominant and aligned with the notch stress**
- **Note: $\sigma_{rad} = 0$ but $\sigma_z \neq 0$**

Typical Loaded Hole



- Max Stress Location Depends upon Where Bearing Force is Headed
  - Axial
  - Shear
- Residual stresses not in phase
- Superposition may not hold

Loaded / Residual Surface Stresses (Element at Corner of Hole)



- Residual Stress Field
- Do Applied Principle Stresses Just Add Together?
  - Must use tensor superposition / resolution to σ_p
  - Watch Princ. Plane Angle

SAE FD&E – Fall 2022: Questions?